PARTIAL DERIVATIVES AS SLOPES OF TANGENT LINES

THE BLUE SURFACE is the GRAPH of $z = f(x,y)$.

$(a,b,0)$ is a point in the xy plane ($z=0$).

Point $P(a,b,c)$ has $c = f(a,b)$.

Curve C_1 consists of the points on the intersection of the surface graph with the vertical plane $y=b$

T_1 is the tangent line, tangent to curve C_1 at the point $P(a,b,c)$.

The slope of the tangent line T_1 is

$$f_x(a,b) = \frac{\partial z}{\partial x}\Big|_{\substack{x=a \\ y=b}}$$

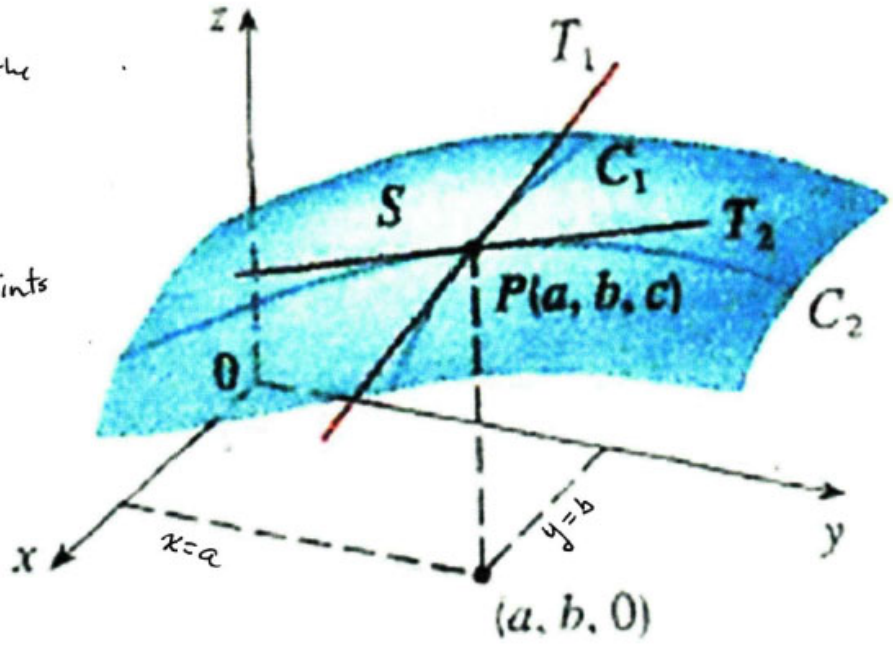

FIGURE 1
The partial derivatives of f at (a, b) are the slopes of the tangents to C_1 and C_2.

C_2 consists of the points on the intersection of the surface graph with the vertical plane $x=a$.

T_2 is the tangent line, tangent to curve C_2 at the point $P(a,b,c)$.

The slope of the tangent line T_2 is

$$f_y(a,b) = \frac{\partial z}{\partial y}\Big|_{\substack{x=a \\ y=b}}$$